REPORT 1388-21

REPORT
by
THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION
COLUMBUS, OHIO 43212

Sponsor	National Aeronautics and Space Administration Office of Grants and Research Contracts Washington, D.C. 20546
Grant Number	NsG-213-61
Investigation of	Theoretical and Experimental Analysis of the Electromagnetic Scattering and Radiative Properties of Terrain, with Emphasis on Lunar-like Surfaces
Subject of Report	The Principle and Design of A Vernier FM Radar System
Submitted by	Tsutomu Suzuki Antenna Laboratory Department of Electrical Engineering
Date	31 May 1966

ABSTRACT

This paper describes the principle and the signal design of a proposed new FM radar system. In order to measure the surface characteristics of a small target at a long distance, or to discriminate among multiple-targets, very accurate range or doppler resolutions are necessary. The proposed system satisfies the range resolution requirement by detecting the target with two different resolutions: coarse resolution for measuring range, and fine resolution for measuring the target details.

The principal advantage of the system is in the Vernier scale for the measurement of a distance. The system is just as easily realiable as conventional FM radar, requires no special filters in the receiver, and represents a saving in the required bandwidth for the same range resolution.

TABLE OF CONTENTS

THE PRINCIPLE AND DESIGN OF A VERNIER FM RADAR SYSTEM

I. INTRODUCTION

The conventional FM radar transmits a signal in which frequency varies linearly with time, and processes the received signal to derive the range (τ) from the frequency difference f_τ between the transmitted and received signal. The range resolution $\Delta\tau$ is limited by the minimum detectable frequency difference Δf_{min} in the receiver. In turn, Δf_{min} is limited by the stability of the local oscillator, the sensitivity of the detector, the bandwidth of the receiver, and the stability of the transmitted frequency. To obtain high range resolution for finite Δf_{min}, the gradient of the frequency shift, $\tan\theta = K$, (see Fig. 1) must be large. The period of the frequency shift T_1 is determined by the maximum distance to the target. Since the output efficiency, to be defined below, is directly dependent on T_1, T_1 should be substantially longer than the time delay at maximum range; τ_{max}. Thus large frequency shift (i.e., system bandwidth) is necessary for an accurate, long range FM radar.

In this paper, a Vernier FM signal (see Fig. 2) is proposed, which can achieve extremely high range resolution with relatively modest system bandwidth. The complete frequency characteristic is a combination of coarse and fine (Vernier) FM shifts, in which the ultimate range resolution is determined by the gradient $\tan\theta_2$ of the fine shift. Furthermore, by a suitable design of local oscillator waveforms, the fine and coarse components of the received signal may be detected separately, and the design of the system can be as straightforward as a conventional FM radar.

II. PRINCIPLE OF THE VERNIER FM RADAR

The frequency relationship between the transmitted and received signals is shown in greater detail in Fig. 3. Here the transmitted frequency shift $f_1(t) + f_2(t)$ is the sum of the coarse, $f_1(t)$, and the fine, $f_2(t)$ frequency shifts, while the received signal is $f_1(t-\tau) + f_2(t-\tau)$. It will be seen that during certain intervals, in particular these marked II, IV, XXI, and XXIII, the differences between the transmitted and received frequencies are constants. To minimize the intervals during which the frequency differences are not constant, and to discriminate between the coarse and fine FM components, consider first the output of a mixer with these two signals as inputs. The output of the mixer will contain the sum

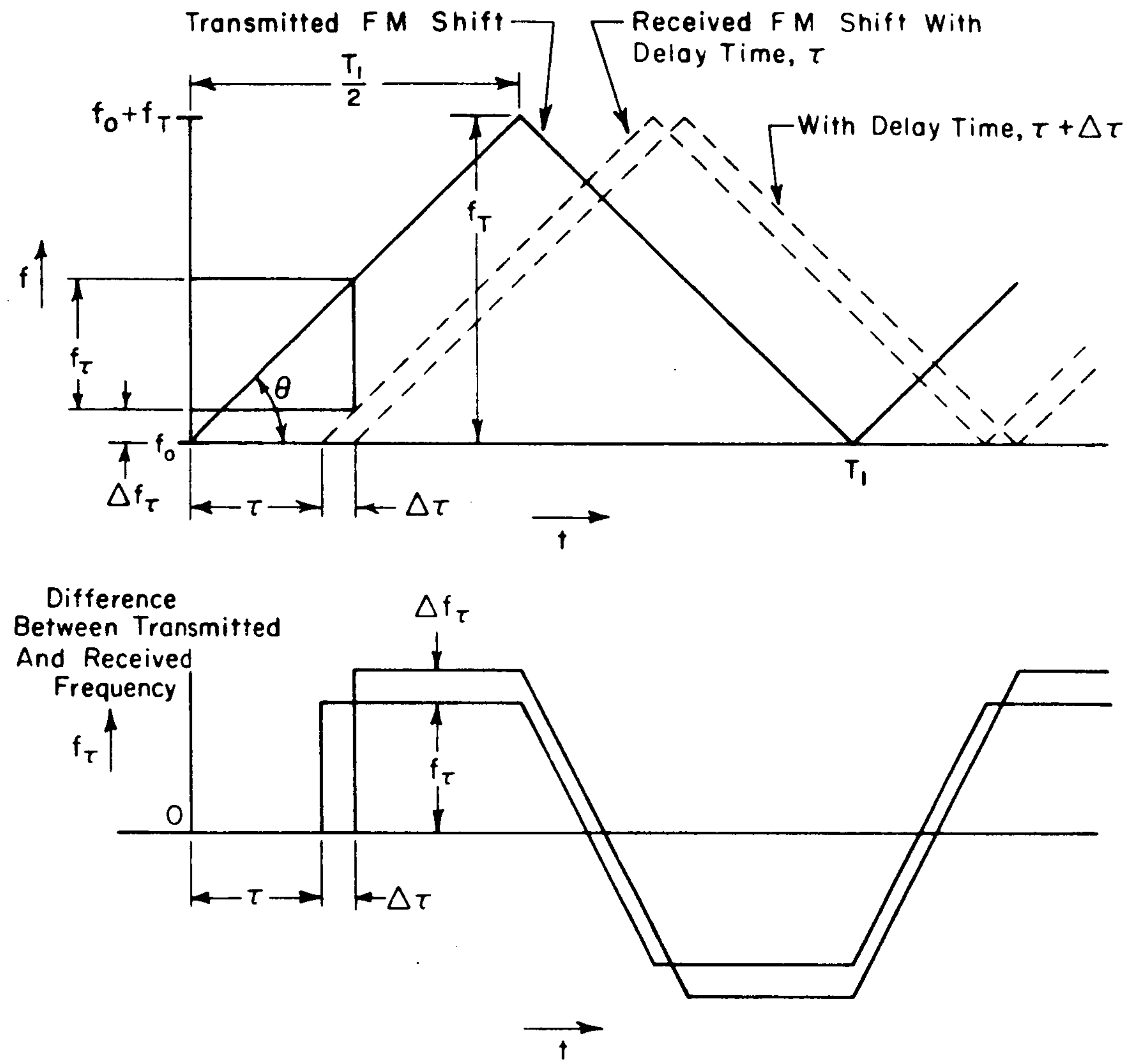


Fig. 1. A conventional linear FM shift.

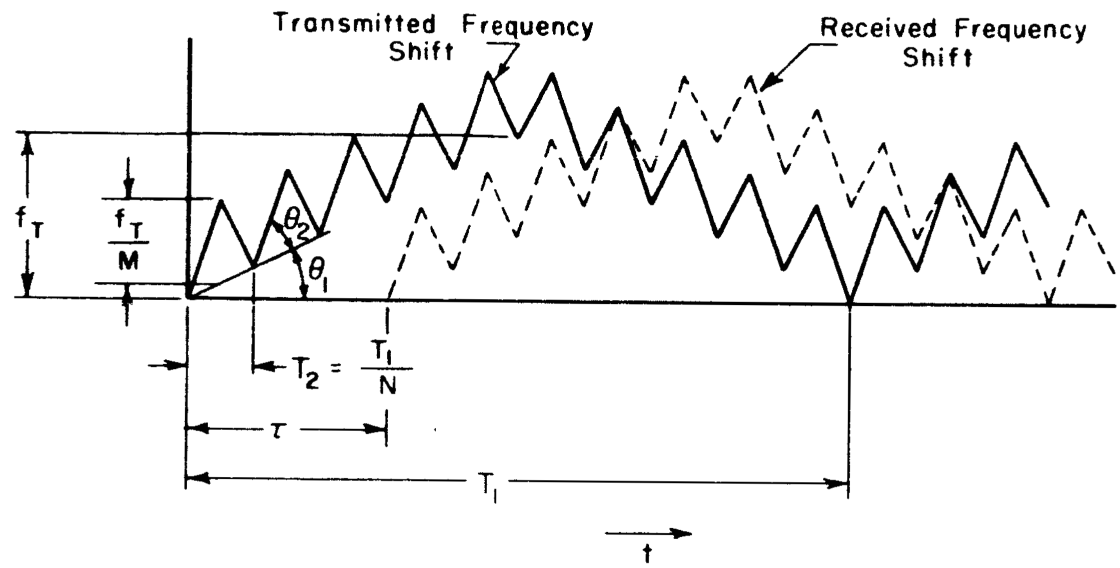


Fig. 2. The frequency shift of the vernier FM shift.

(1) $$f_1(t) + f_2(t) + [f_1(t-\tau) + f_2(t-\tau)]$$

and the difference

(2) $$f_1(t) + f_2(t) - [f_1(t-\tau) + f_2(t-\tau)]$$

frequencies, of which the difference component only, Eq. (2), may be selected by a suitable filter. Next, suppose the transmitter frequency had been of the form $[-f_1(t) + f_2(t)]$; the difference output of the mixer would be

(3) $$[-f_1(t) + f_2(t)] - [-f_1(t-\tau) + f_2(t-\tau)] \quad .$$

If now signals Eqs. (2) and (3) become the inputs to a third mixer, its output will have a sum component

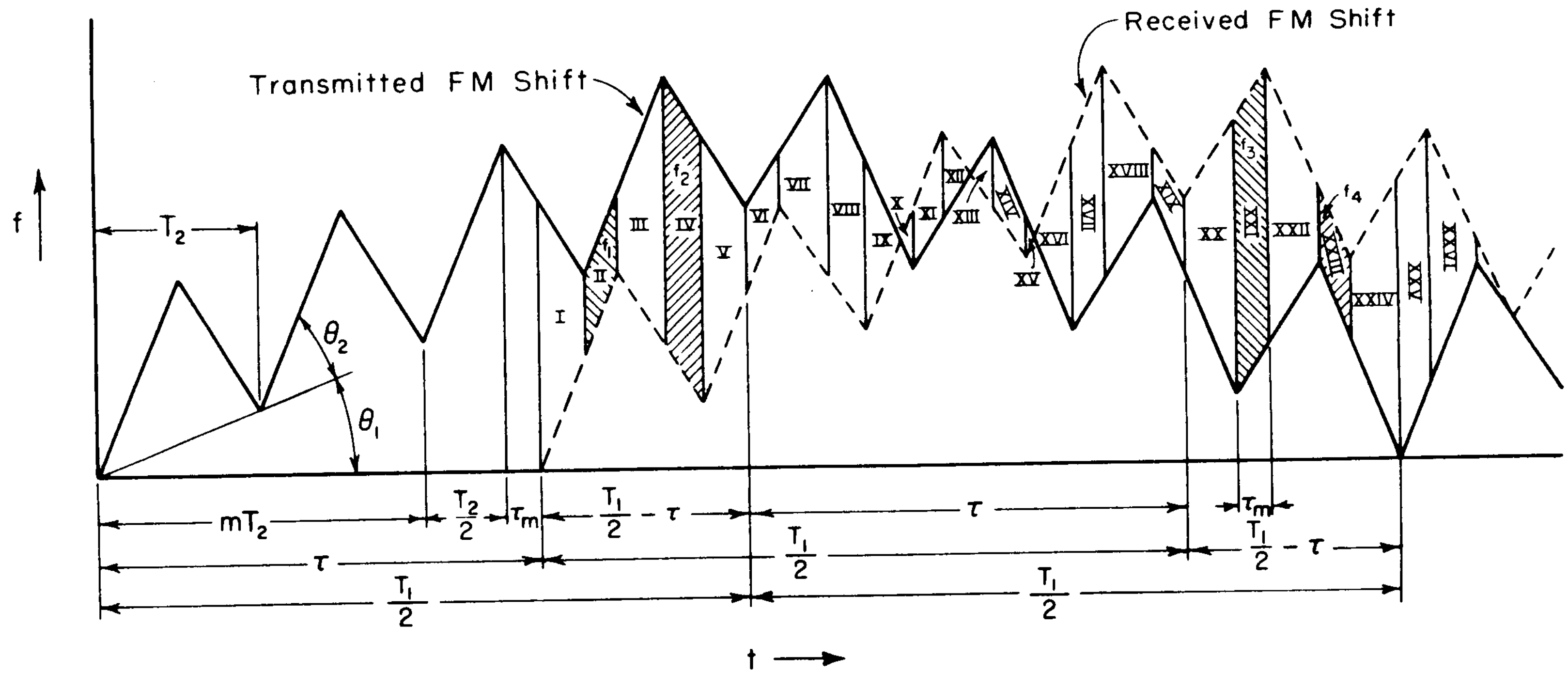


Fig. 3. The frequency shifts of a transmitted signal and the received signal.

(4) $$2\left|f_2(t) - f_2(t-\tau)\right|$$

involving only the fine shift, and a difference component

(5) $$2\left|f_1(t) - f_1(t-\tau)\right|$$

involving only the coarse shift. Relations 4 and 5 are derived explicitly in the Appendix.

Figure 4 shows schematically how this separation into the two components, coarse and fine, may be made by a special choice of local oscillator waveforms, even though only one signal $f_1(t) + f_2(t)$ is actually transmitted. In Fig. 4a is shown the transmitted waveform; Fig. 4b shows the received waveform, and the first L.O. signal, a reproduction of the transmitted signal delayed by a certain interval τ_0; Fig. 4c shows the second L.O. signal, a reproduction of the transmitted signal delayed by $\tau_0 + T_1/2$, $T_1/2$ being the half period of the coarse frequency shift. Note in Figs. 4b and 4c that the desired intervals of constant frequency difference are shaded. The actual frequency differences corresponding to Figs. 4b and 4c are shown in Figs. 4d and 4e. Note also that this procedure effectively reverses the sign of the coarse FM only, i.e., $f_1(t)$ and $f_1(t-\tau)$ both change sign. The last two parts, Figs. 4f and 4g show the difference and the sum, respectively of the frequencies of Figs. 4d and 4e; they correspond to the desired coarse FM signal (Eq. (5)) and the fine FM signal (Eq. (4)).

A block diagram of a receiver to implement this detection process is shown in Fig. 5. Apart from the double local oscillators, it is similar to a conventional FM radar system, and amenable to the same design techniques.

III. SYSTEM CONSIDERATIONS

This section deals with some considerations of signal and system design.

A. Choice of the Signal Parameters, T_1, T_2, K_1, K_2 and f_T

A number of constraints on the system parameters are imposed by the desired performance specifications for the system. These are most easily described in the limit $\tau_0 \to 0$.

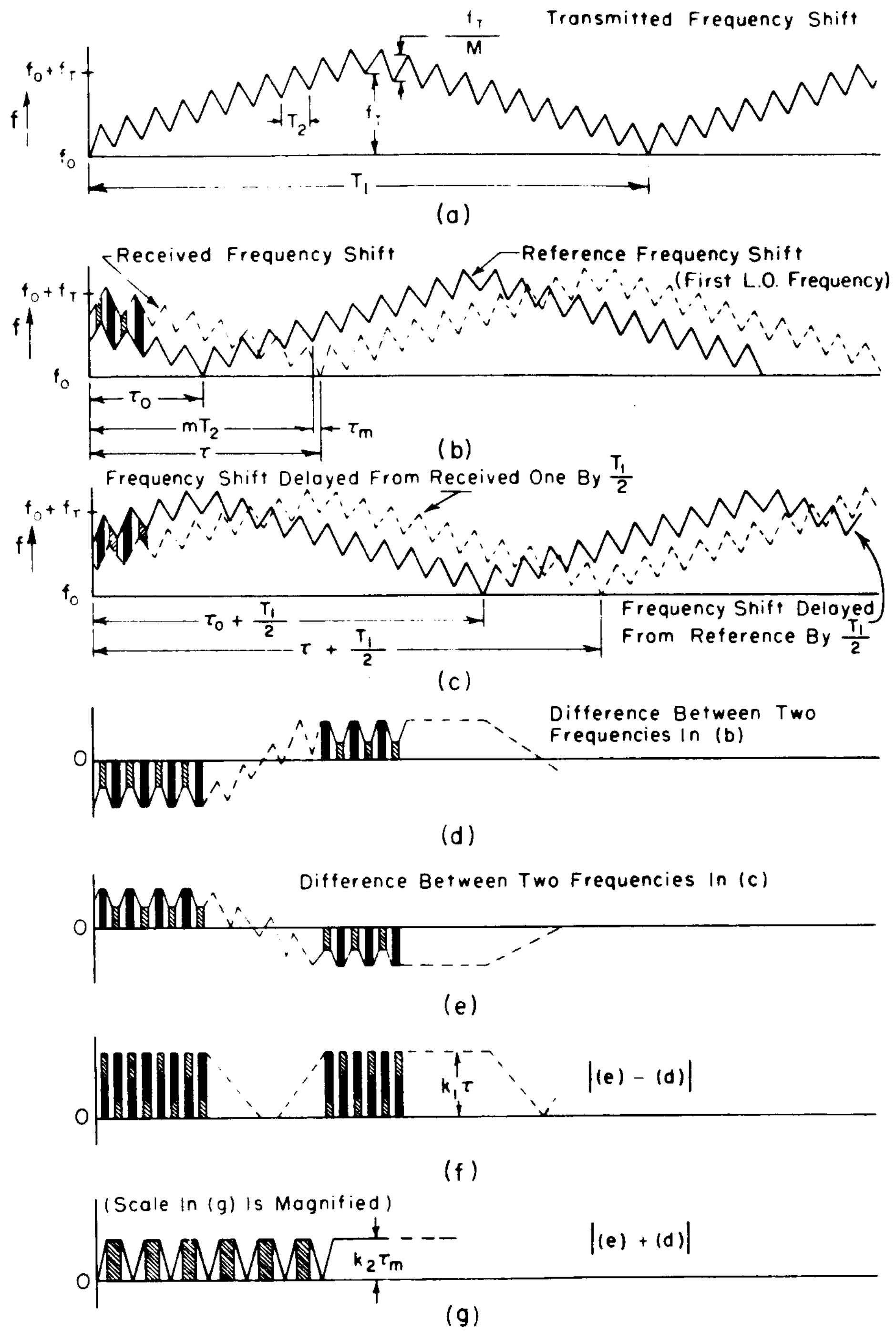


Fig. 4. Separation of the fine and the coarse FM shifts of the veriner signal.

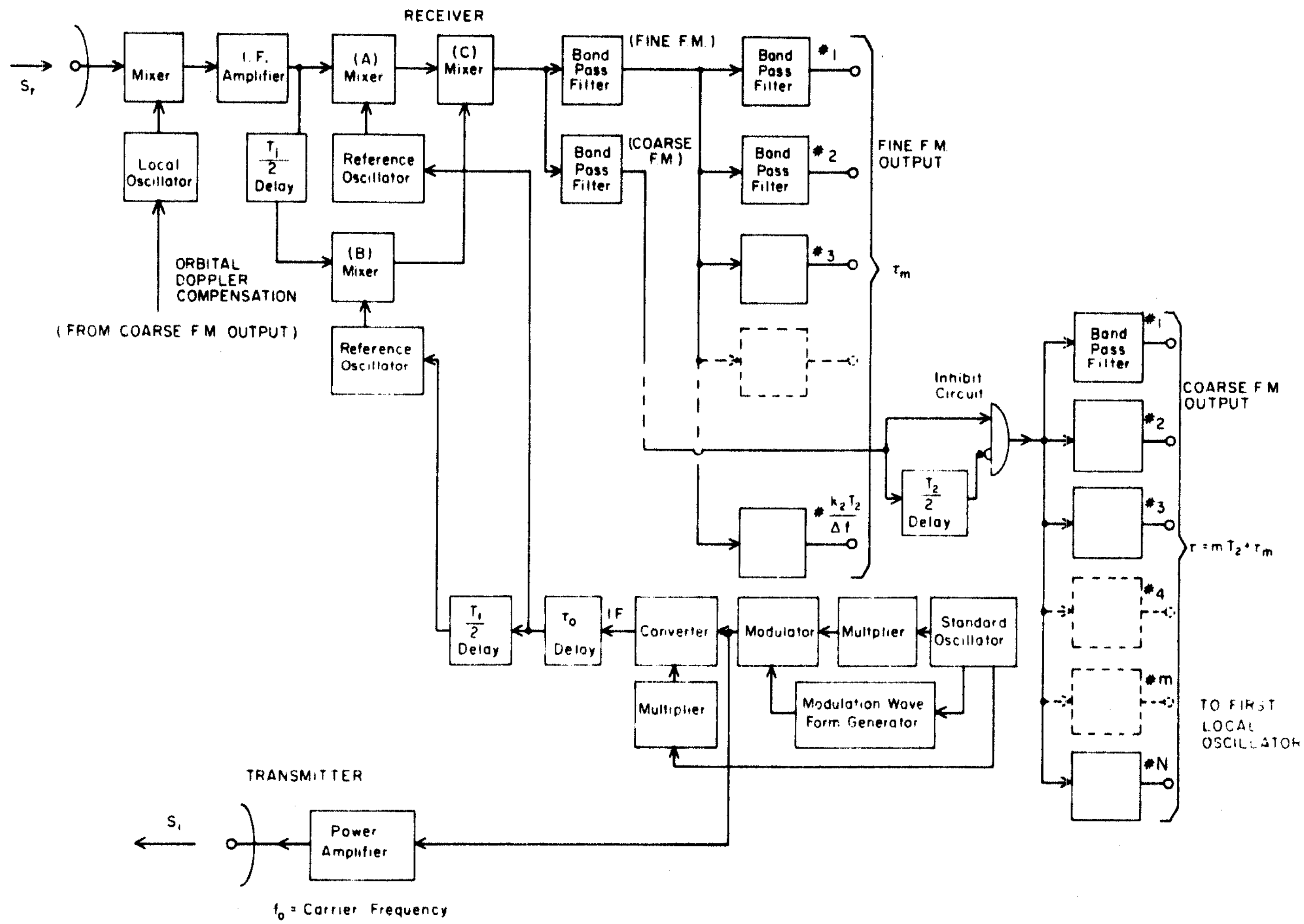


Fig. 5. Simplified block diagram of the vernier FM radar.

The range resolution ΔR, the minimum detectable delay time, $\Delta\tau_{min}$, and the minimum detectable frequency change Δf_{min} are related by

$$(6) \qquad \Delta f_{min} = K_2 \, \Delta\tau_{min} = K_2 \left(\frac{2}{C}\right) \Delta R$$

where C is the velocity of light and K_2 is the fine FM sweep rate.

If the frequency stability (S) of the transmitter, is defined as $S = df/f_o$ (where df is the maximum frequency drift in either the transmitter or the L.O. signals during the interval T_1, and f_o is the carrier frequency) then, clearly, Δf_{min} cannot be less than the oscillator drift $\Delta f_{min} \geq df$. Combining the last three relations gives a lower limit to the fine FM sweep rate,

$$(7) \qquad K_2 \geq \frac{S f_o C}{2\Delta R} \quad .$$

If the transmitted and received frequencies overlap, (see Fig. 6a) discrimination between coarse and fine components is difficult (see Appendix). To avoid this, the minimum delay time τ_{min}, determined by the minimum desired range R_{min}, should satisfy $K_1 \tau_{min} \geq K_2 T_2/2$, which sets a lower limit on the coarse FM sweep rate

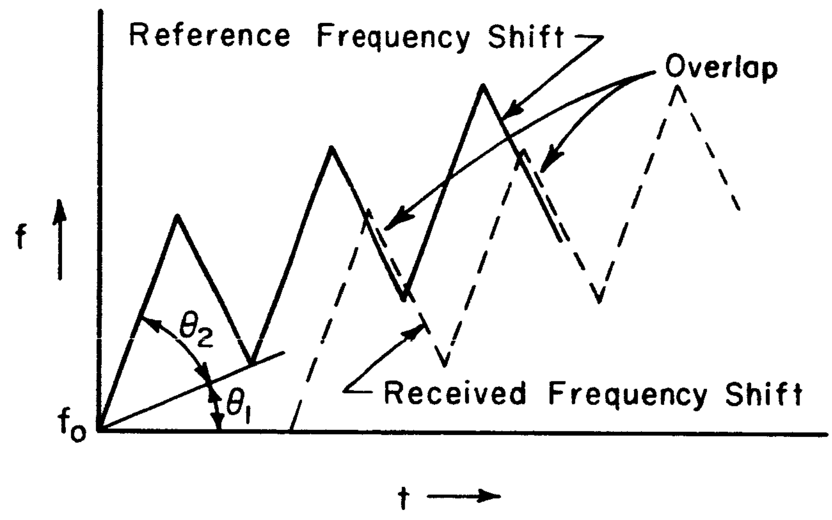
$$(8) \qquad K_1 \geq \frac{K_2 \, T_2}{2\tau_{min}} = \frac{K_2 \, T_2 \, C}{4 \, R_{min}} \quad .$$

Also, since the resolution of the coarse FM need be no better than $T_2/2$, one has $K_1(T_2/2) \geq \Delta f_{min}$ which, together with the previous relations, implies an approximate value for T_2,

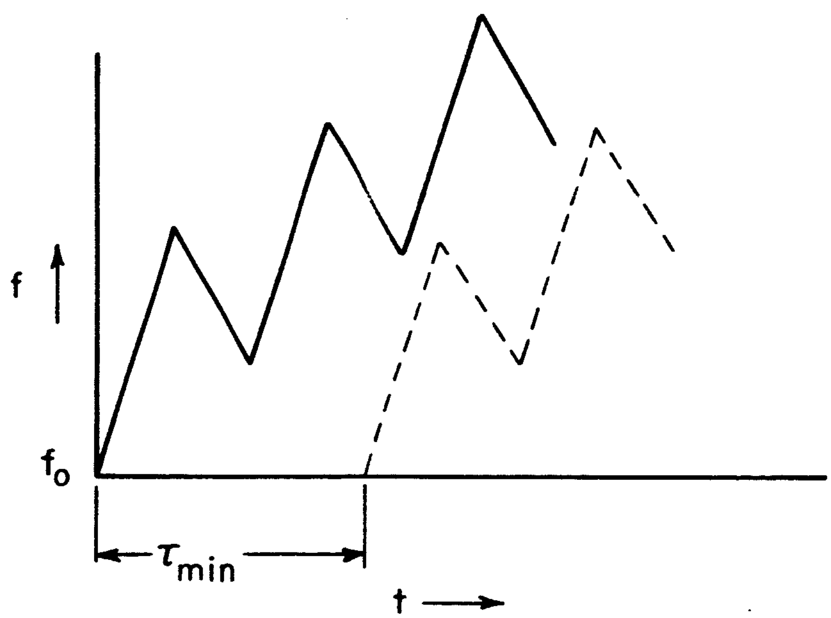
$$(9) \qquad T_2 \simeq 2 \sqrt{\frac{S f_o \tau_{min}}{K_2}} \quad .$$

The time delay τ, and the period T_1 determine the efficiency η of the receiver output (see Fig. 7), defined here as

$$(10) \qquad \eta = \left(\frac{T_1}{2} - \tau\right) \left(\frac{T_1}{2}\right)^{-1} = 1 - \frac{2\tau}{T_1} \quad ,$$



(a) Overlapping Signals



(b) Separated Signals

Fig. 6. Separation between the received frequency shift and the reference shift.

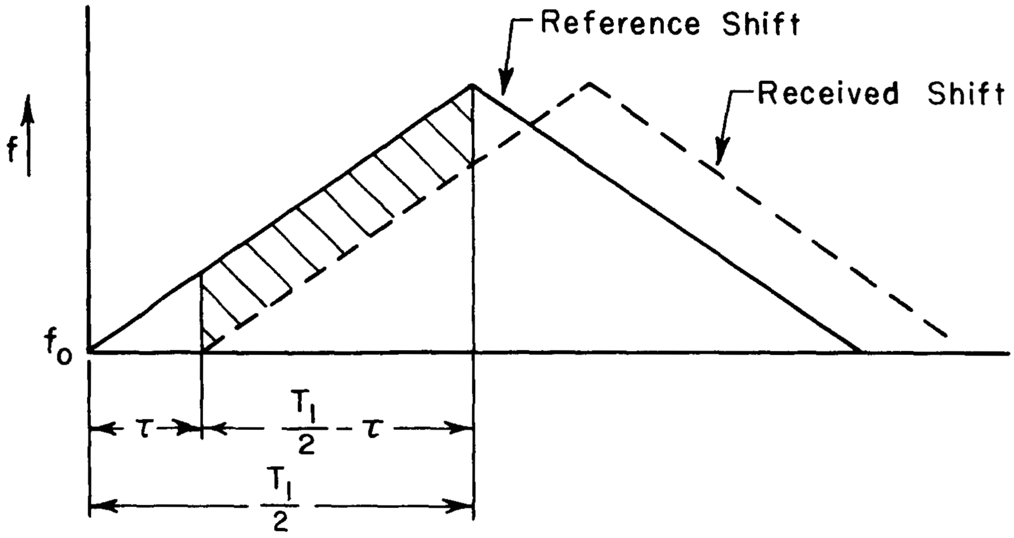


Fig. 7. The output efficiency η.

i.e., the ratio of the duration of usable signal to the period of the coarse FM shift. Since the lowest efficiency occurs at maximum delay, $\eta_{min} = 1 - 2\tau_{max}/T_1$, the period T_1 is given by the minimum acceptable efficiency η_{min} and the maximum design range R_{max}, viz. $T_1 = (1-\eta_{min})^{-1} (R_{max}/C)$.

B. Blind Ranges

If the target distance is $(CT_2/2)(P + 1/2)$ (P an integer) the output of the fine FM does not contain a constant frequency difference. To eliminate these "blind ranges", which correspond to "blind delays" $\tau_B = T_2(P + 1/2)$, an additional delay of $T_2/2$ is introduced into the reference signal. Figure 8a shows the "blind range" condition, Fig. 8b shows an appropriate circuit to implement the extra delay $T_2/2$ and Fig. 8c shows the final sum and difference frequencies. It will be seen that the output efficiency (at the blind range) is 100% for the delayed reference, and 0% for the original reference. Thus by combining the outputs of the original and the $T_2/2$ delayed reference circuits, there will always exist the required intervals of constant frequency difference. The additional circuitry not only eliminates the blind ranges, but increases the output efficiency of the fine FM at all ranges.

C. System Bandwidths

The spectral bandwidth for the coarse and fine signals are larger than the actual frequency excursions $\omega_T = 2\pi f_T$ and $\omega_T/M = 2\pi f_T/M$, because these waveforms are periodic with periods T_1 and T_1/N respectively. (Here the ratio $\sigma = K_2/K_1 = N/M$ gives the improvement in range resolution over a conventional FM system with the same maximum range and maximum frequency excursion. Clearly, a worthwhile vernier system will have $\sigma >> 1$.) To obtain the actual spectra $E_i(\omega)$ for the coarse signal,

(11) $$e_1(t) = \cos\left(\frac{\omega_T}{T_1} t^2 + \omega_o t + C\right), \qquad -\frac{T_1}{2} \leq t < \frac{T_1}{2} \quad ,$$

and the fine signal

(12) $$e_2(t) = \cos\left(\frac{\omega_T}{MT_1} t^2 + \omega_o t + C'\right), \qquad -\frac{T_1}{2N} \leq t < \frac{T_1}{2N} \quad ,$$

(with C, C' constants and e_1, e_2 both zero outside the above intervals) one has

(13) $$E_i(\omega) = 2\int_0^{\infty} e_i(t)\, e^{-j\omega t}\, d\omega \quad i = 1, 2. \qquad \text{(see Fig. 9)}$$

These may be evaluated in terms of the Fresnel integrals:[3]

(14) $$E_1(\omega) = 2\int_0^{T_1/2} e_1(t)\, \epsilon^{-j\omega t}\, dt \;,$$

$$= \left(\frac{\pi T_1}{2\omega_T}\right)^{\frac{1}{2}} \left\{ \epsilon^{j\delta_1} \int_{y_1}^{y_3} \epsilon^{-j\pi y^2/2}\, dy + \epsilon^{-j\delta_2} \int_{y_2}^{y_4} \epsilon^{j\pi y^2/2}\, dy \right\} ,$$

$$= \left(\frac{\pi T_1}{2\omega_T}\right)^{\frac{1}{2}} \{\epsilon^{j\delta_1}[(C(y_3) - C(y_1)) - j(S(y_3) - S(y_1))]$$

$$+ \epsilon^{-j\delta_2}[(C(y_4) - C(y_2)) + j\,(S(y_4) - S(y_2))]\} \;,$$

where C(y) and S(y) are

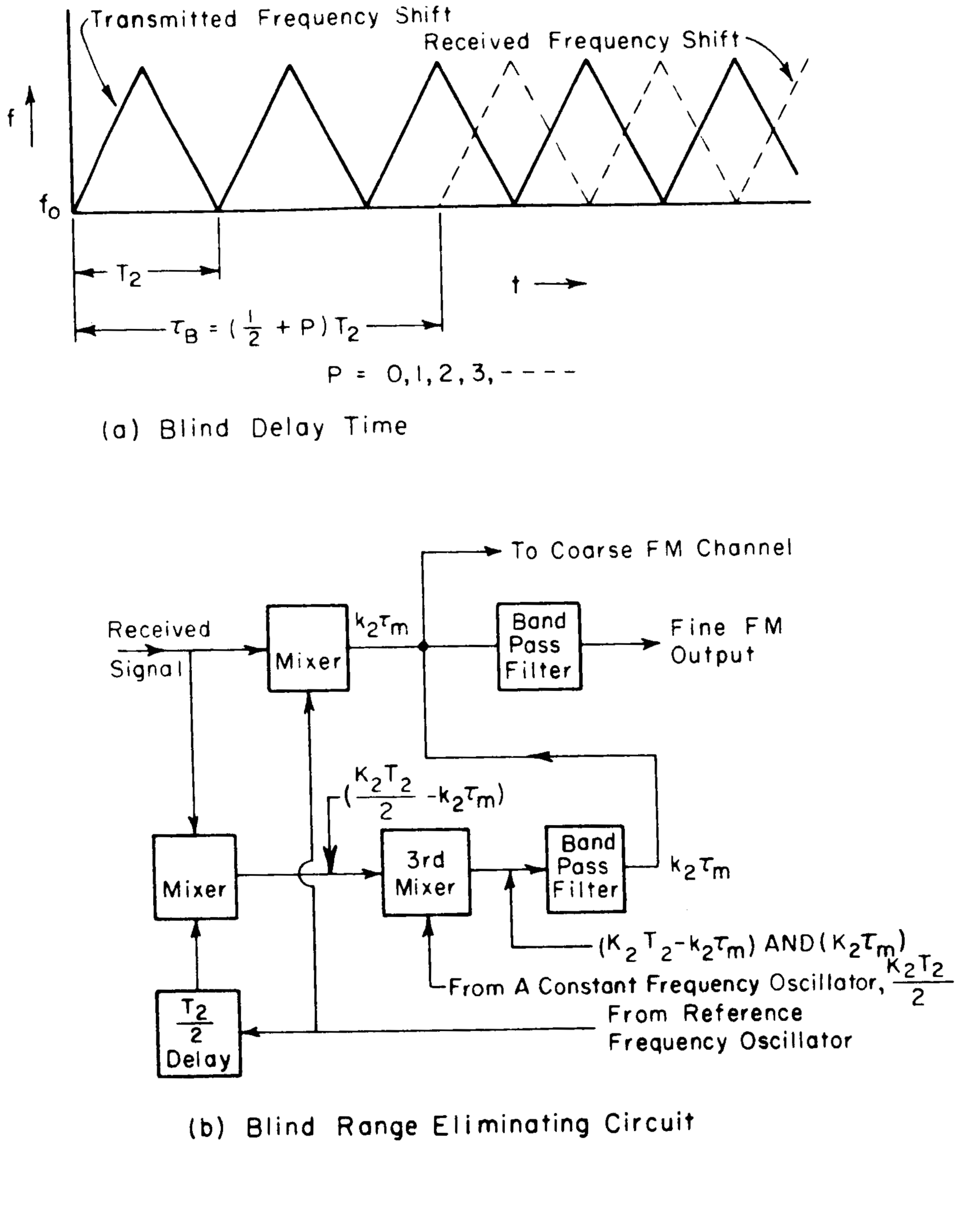
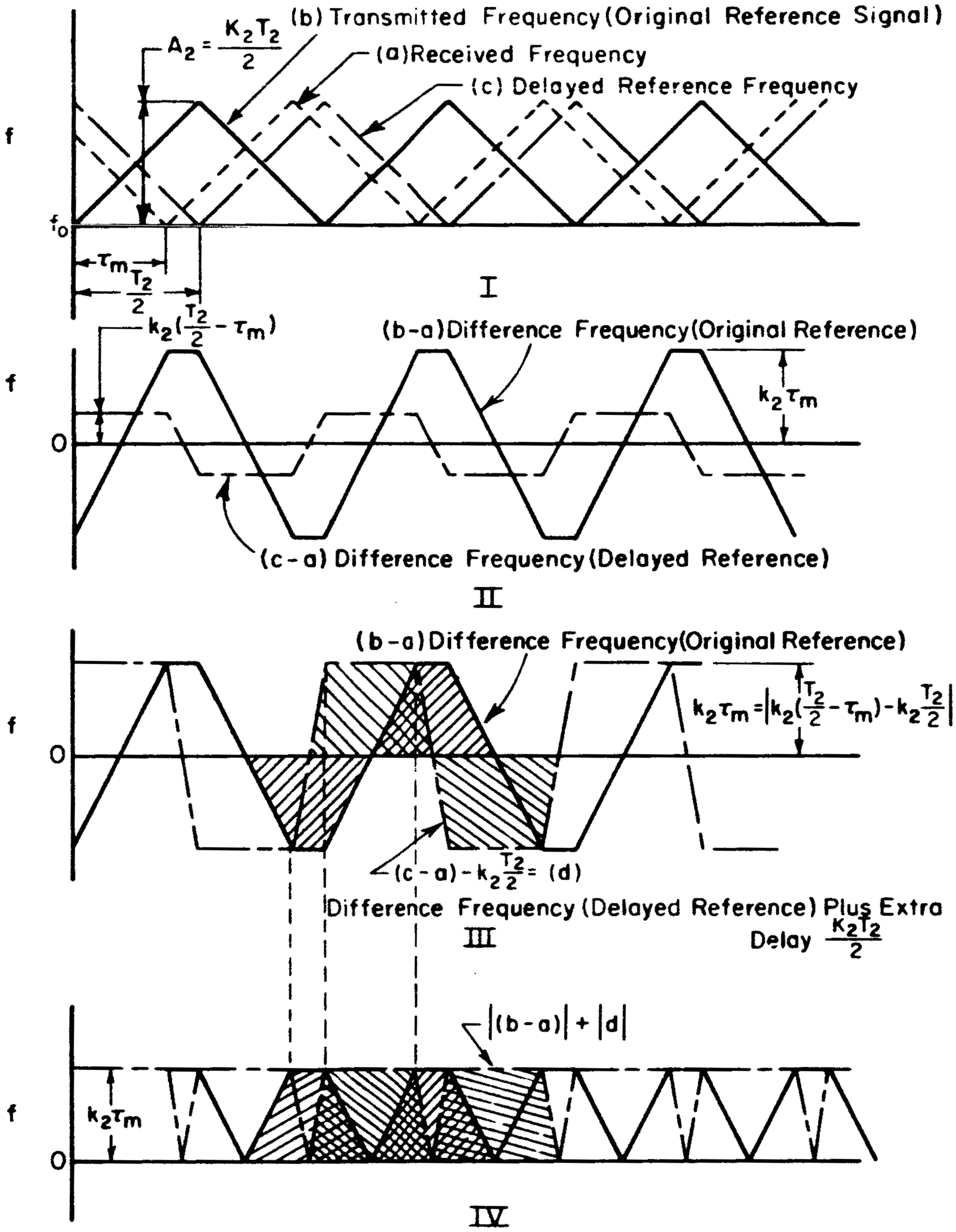


Fig. 8. Blind ranges and their elimination.



(c) Elimination Of Blind Delay Time

Fig. 8. Blind ranges and their elimination. (Continued)

$$C(y) = \int_0^y \cos \frac{\pi}{2x^2} dx$$

and

$$S(y) = \int_0^y \sin \frac{\pi}{2x^2} dx,$$

and

$$\delta_1 = T_1(\omega_o + \omega)^2 (4\omega_T)^{-1} - C,$$

$$\delta_2 = T_1(\omega_o - \omega)^2 (4\omega_T)^{-1} - C ,$$

$$y_1 = (\omega_o + \omega) (T_1/2\pi\omega_T)^{\frac{1}{2}} ,$$

$$y_2 = (\omega_o - \omega) (T_1/2\pi\omega_T)^{\frac{1}{2}}$$

$$y_3 = (\omega_o + \omega_T + \omega)(T_1/2\pi\omega_T)^{\frac{1}{2}}$$

$$y_4 = (\omega_o + \omega_T - \omega) (T_1/2\pi\omega_T)^{\frac{1}{2}}$$

Similarly,

(15) $$E_2(\omega) = 2 \int_0^{T_1/2N} e_2(t) \epsilon^{-j\omega t} dt .$$

The forms of the two spectra, examples of which are shown in Fig. 10, are quite similar except for the scale factor $(M/N)^{\frac{1}{2}}$. To estimate the bandwidth, note that $C(y)$ and $S(y)$ have essentially reached their limiting values of 1/2 when $y > 3$. Thus $E_1(\omega)$ will contain very little power for angular frequencies less than

(16) $$\omega_1 = 3\left(\frac{2\pi\omega_T}{T_1}\right)^{\frac{1}{2}} = 3(\pi K_1)^{\frac{1}{2}} ,$$

so that the bandwidth of the coarse FM signal is approximately (see Fig. 10)

(17) $$B\bar{\omega}_1 = \omega_T + 6(\pi K_1)^{\frac{1}{2}} ,$$

similarly,

(18) $$B\bar{\omega}_2 = \left(\frac{\omega_T}{M}\right) + 6(\pi K_1\sigma)^{\frac{1}{2}} = \frac{\omega_T\sigma}{N} + 6(\pi K_1\sigma)^{\frac{1}{2}} .$$

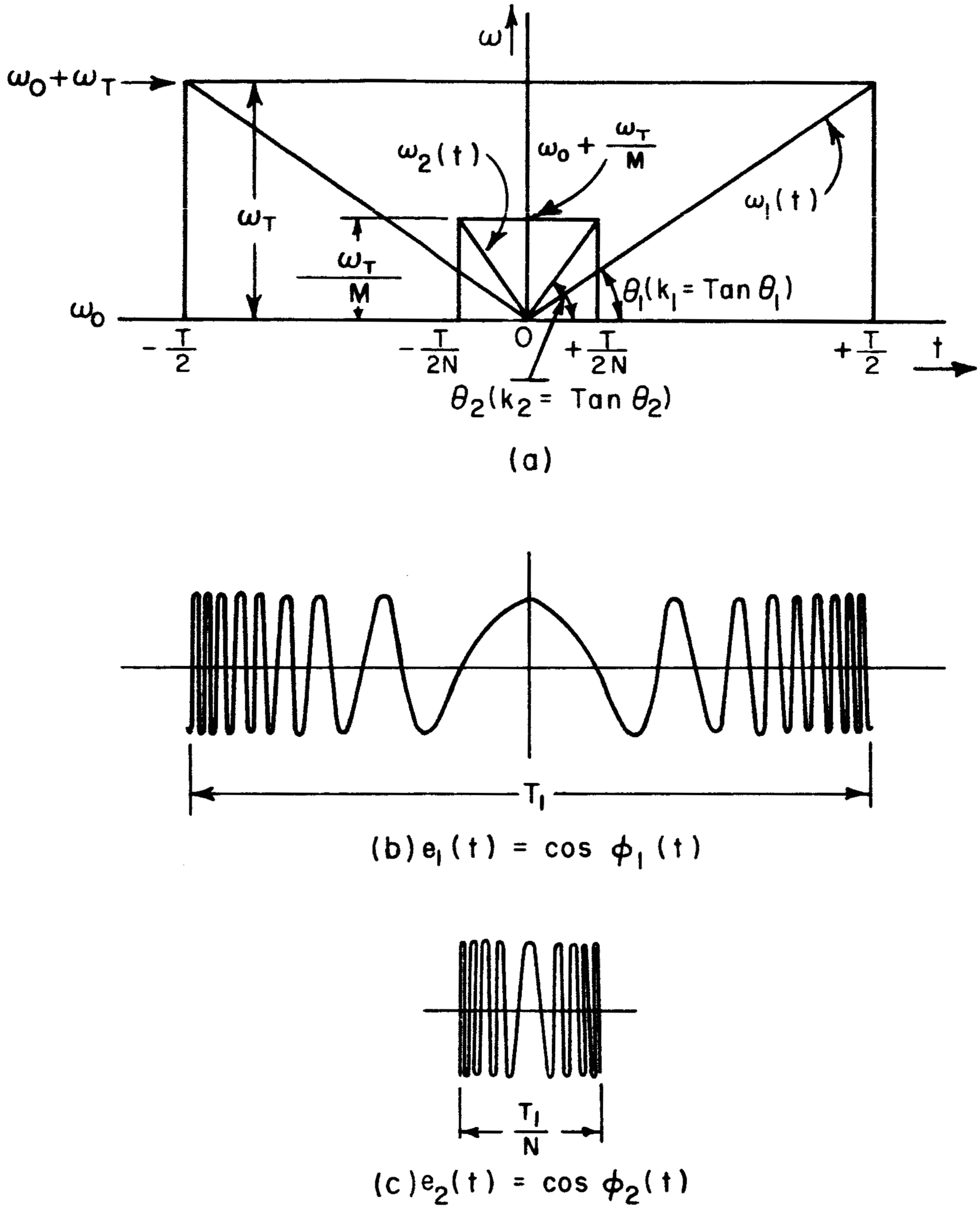


Fig. 9. Two FM shifts, $e_1(t)$ and $e_2(t)$.

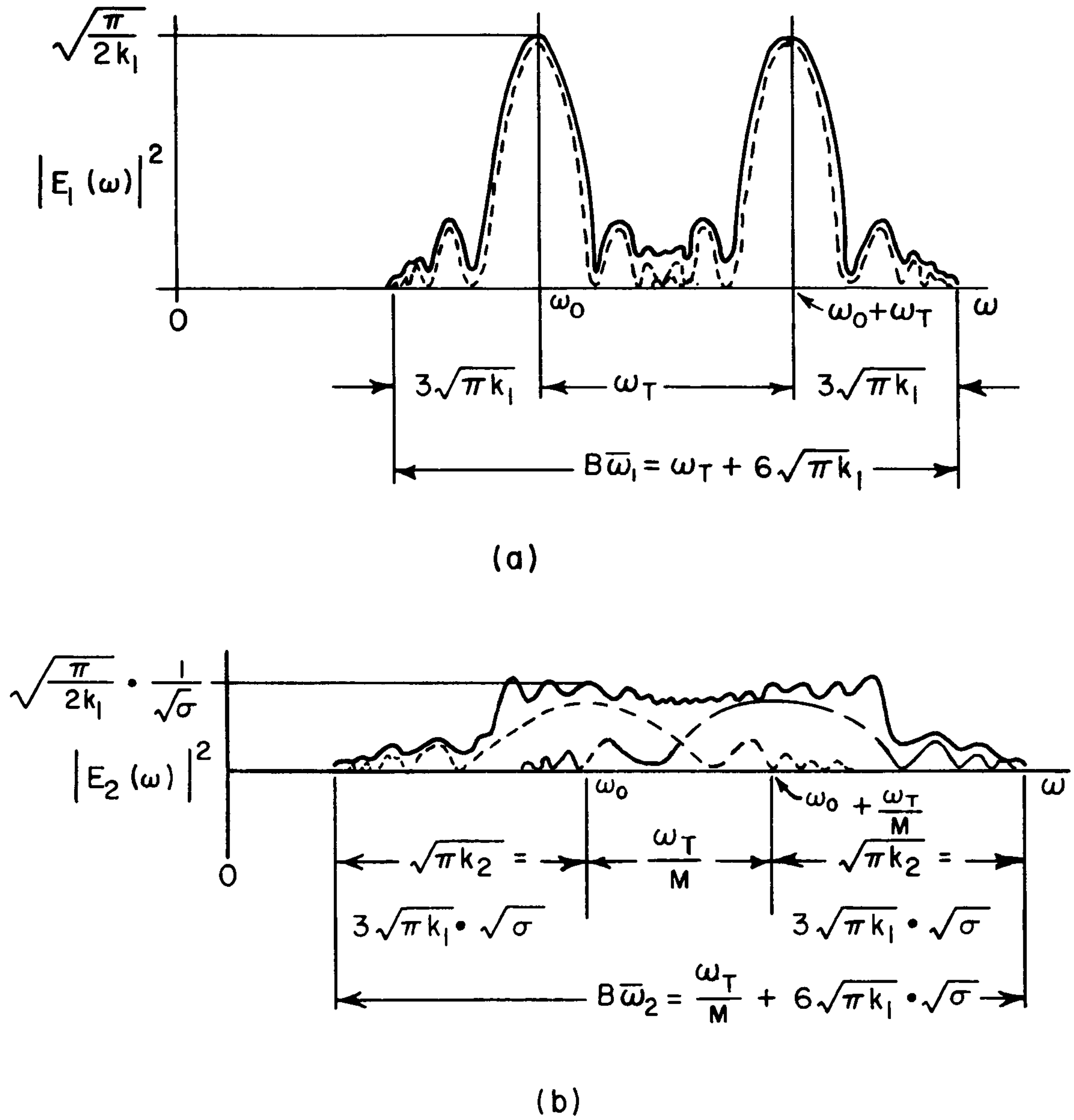


Fig. 10. Approximate spectra, $E_1(\omega)$ and $E_2(\omega)$.

The bandwidth of the vernier FM signal may be determined from its spectrum, $E_3(\omega)$ which is the convolution (*) of $E_1(\omega)$ and $E_2(\omega)$,

$$(19) \qquad E_3(\omega) = \int_{-\infty}^{\infty} (e_1(t) \cdot e_2(t))\, e^{-j\omega t}\, dt = E_1(\omega) * E_2(\omega) \ .$$

Figure 11 shows the approximate form of $E_3(\omega)$. It can be seen that the bandwidth of the vernier signal is approximately the same as that of the fine signal if σ is large.

The bandwidth of a conventional FM signal which has the same range resolution as the fine FM, and the same maximum range as the coarse FM, is

$$(20) \qquad B\bar{\omega}_3 = \frac{K_2 T_1}{2} + 6(\pi K_2)^{\frac{1}{2}} = \omega_T \sigma + 6(\pi K_2)^{\frac{1}{2}} \ .$$

Assuming that the bandwidth of the vernier FM signal is equal to that of the fine FM, the ratio of the bandwidth of the vernier FM signal to that of the conventional FM signal is

$$(21) \qquad \gamma = \left(\frac{\omega_T \sigma}{N} + 6(\pi K_2)^{\frac{1}{2}}\right) (\omega_T \sigma + 6(\pi K_2)^{\frac{1}{2}})^{-1} \ .$$

It is always seen that the ratio γ is less than unity.

D. Doppler Shift

The doppler shift in the vernier FM Radar is approximately the same as that of the conventional FM Radar. For the detection of the doppler shift by this system, the same techniques as the conventional one are also applicable. These will not be considered in this paper.

E. Discrimination of the Difference Between the Coarse and the Fine FM, and Suppression of Undesired Harmonic Components

It is desired to separate the fine and coarse FM components by simple filtering. If the system is designed so that the fundamental frequency of the

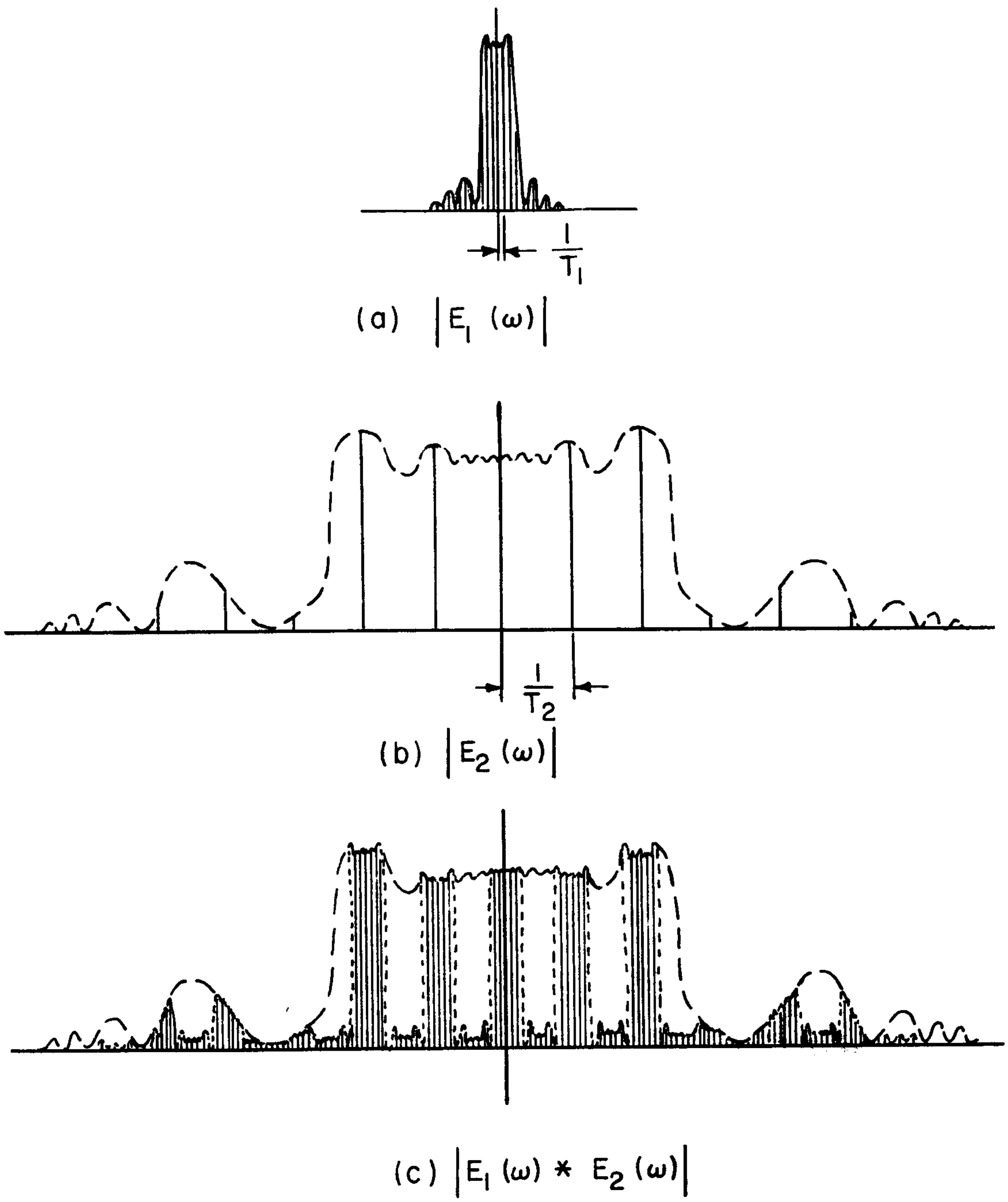


Fig. 11. Approximate spectrum of the vernier FM signal.

output of the fine FM signal is always less than $k_2(T_2/2)$, and that of the coarse FM signal is always between $k_2(T_2/2)$ and $2k_2(T_2/2)$, the separation of two outputs (fine and coarse) is very simple.

The above restriction for the fine FM is easily realizable by satisfying the condition shown in Eq. (8), but the one for the coarse FM is not always realizable because it is closely related to the range of the target, which is not fixed in space. Therefore, some harmonics of the fundamental frequency of the fine FM signal may possibly appear in the coarse FM output. These undesired frequency components can be eliminated by using an equivalent comb-filter circuit, as shown in Fig. 12; however, another blind range in the coarse FM output may occur.

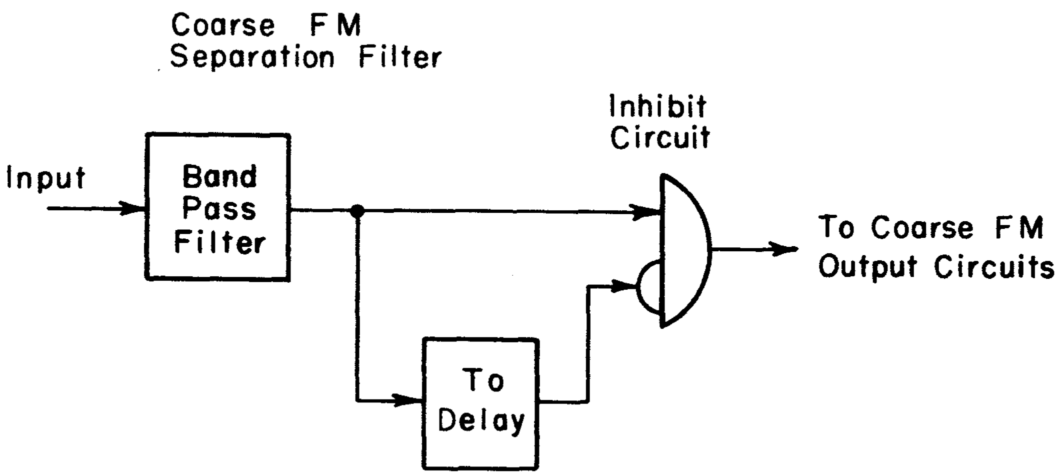


Fig. 12. The equivalent comb-filter circuit which eliminates the harmonic frequencies of $1/T_1$.

F. The Efficiency of the Fine FM Output

From Eq. (9) we can determine the period T_2 of the fine FM. Good resolution in the fine FM output requires narrow bandwidth in the output filter. A filter has a transient response to the incoming signal. The build-up time in the output of the filter is approximately proportional to the reciprocal of the bandwidth of the filter. Therefore, high range-resolution which results from using a very narrow bandwidth filter decreases the efficiency of the fine FM output if the duration of the fine FM signal is very small. Thus, a filter which has a short transient time and narrow bandwidth is desirable for both high output efficiency and resolution.

IV. AN EXAMPLE OF THE CALCULATION OF THE SIGNAL PARAMETERS

An example of signal design for detection of the surface characteristics of a target (such as an Echo satellite) will now be discussed. For this example the parameters are chosen as shown in Table I:

TABLE I

Descriptions	Notations	Values
required range resolution	ΔR	2 m
carrier frequency of the transmitted signal	f_o	1,000 MHz
frequency stability of f_o	$S = \frac{df}{f_o}$	10^{-7}
minimum detectable range	D_{min}	500 N.M.
maximun detectable range	D_{max}	1,000 N.M.
minimum efficiency of the coarse FM output	η_{min}	0.5
gradient of the fine FM shift	K_2	$3/4 \cdot 10^{10}$
gradient of the coarse FM shift	K_1	$1.9 \cdot 10^{7}$
period of the fine FM shift	T_2	17 μs
period of the coarse FM shift	T_1	12 ms
resolution improvement factor	$\sigma = K_2 / K_1$	400
bandwidth required for the fine FM	$B\bar{\omega}_2$	1.04 MHz
bandwidth required for the coarse FM	$B\bar{\omega}_1$	166 KHz

Table I shows that if we try to detect a target with a conventional FM radar with a resolution, $\Delta R = 2m$, minimum detection efficiency, $\eta_{min} = 0.5$ and maximum range, $R_{max} = 1,000$ N.M., the required bandwidth is

$$B\overline{\omega_3} = \omega_T \sigma + 6(\pi K_2)^{\frac{1}{2}} \simeq 46 \text{ MHz} . \tag{22}$$

Such a signal, while possible to realize, is not a practical one; the very wide bandwidths necessary both in the transmitter and receiver are highly undesirable. This may be compared with the bandwidth of only 1.04 MHz for the vernier FM system of equivalent performance.

V. CONCLUSIONS

From the preceding discussion, it may be concluded that the vernier FM radar can detect a target with better range resolution and smaller bandwidth than the conventional FM radar. For example, the surface of a target at the range of 1,000 nautical miles can be resolved to 2m, with a bandwidth of only 1.04 MHz.

The doppler resolution of the vernier FM radar is the same as that of the conventional FM radar. This system requires delay lines which are very stable and have long delay times. The characteristics of these delay lines will be one of the important factors in a practical system design.

The biggest advantage of this system is that the minimum range resolution of a target is directly separated from the maximum detectable range of the target.

ACKNOWLEDGEMENTS

The author expresses his appreciations to Drs. J.H. Richmond, W.H. Peake, R.T. Compton, J.W. Eberle, and Mr. S.L. Zolnay for their many helpful comments and suggestions.

REFERENCES

1. Skolnik, M.I., Introduction to Radar Systems, McGraw-Hill Book Co., Inc., New York (1962).

2. Papoulis, A., The Fourier Integral and Its Applications, McGraw-Hill Book Co., Inc., New York, p. 165 (1962).

3. The Mathematical Handbook, Marnzen Co., Tokyo, Japan (1961).

4. Cook, C.E., "A Pulse Compression Predistortion Function for Efficient Sidelobe Reduction in a High-Power Radar," Proc. of the IRE, pp. 377-398, (April 1964).

APPENDIX
(A METHOD FOR SEPARATING THE COARSE AND FINE FM)

$f_1(t)$ is the frequency shift of coarse FM of the transmitted signal:

$f_2(t)$ is the frequency shift of fine FM of the transmitted signal:

$f_1(t-\tau)$ is the frequency shift of coarse FM of the received signal:

$f_2(t-\tau)$ is the frequency shift of fine FM of the received signal:

f_o is the unshifted frequency of the transmitted and received signals:

$[f_o + f_1(t) + f_2(t)]$ is the instantaneous frequency of the transmitted vernier FM signal:

$[f_o + f_1(t-\tau) + f_2(t-\tau)]$ is the instantaneous frequency of the received vernier FM signal.

Assume the mixer has square-law characteristics. By mixing the transmitted and received signals and by filtering out the difference frequency, we obtain the output of mixer (A) (see Fig. 5) as:

$$(1) \qquad f_A = \left|(f_1(t) + f_2(t)) - (f_1(t-\tau) + f_2(t-\tau))\right| ,$$

$$= \left|(f_1(t) - f_1(t-\tau)) + (f_2(t) - f_2(t-\tau))\right| .$$

If the coarse shift components, $f_1(t)$ and $f_1(t-\tau)$, both are reversed in phase, the output of mixer (B) becomes

$$(2) \qquad f_B = \left|(-f_1(t) + f_2(t)) - (-f_1(t-\tau) + f_2(t-\tau))\right| ,$$

$$= \left|-(f_1(t) - f_1(t-\tau)) + (f_2(t) - f_2(t-\tau))\right| .$$

Next, the output of the mixer (C), which has two inputs, f_A and f_B, is

$$(3) \qquad f_C = (f_A + f_B) + \left|f_A - f_B\right| + 2(f_A + f_B) .$$

If the mixer (C) is balanced, the output includes only odd harmonics of the input frequency. Then, the output becomes

(4) $$f_C = (f_A + f_B) + |f_A - f_B| \ .$$

The first term in Eq. (4) is

(5) $$\begin{aligned} f_A + f_B &= |(f_1(t) - f_1(t-\tau)) + (f_2(t) - f_2(t-\tau))| \\ &\quad + |-(f_1(t) - f_1(t-\tau)) + (f_2(t) - f_2(t-\tau))| \\ &= 2|f_2(t) - f_2(t-\tau)| \ , \end{aligned}$$

when both f_A and f_B have the same sign, and $2|f_1(t) - f_1(t-\tau)|$, when the sign of f_A is different from that of f_B. The second term in Eq. (4) is

(6) $$|f_A - f_B| = 2|f_1(t) - f_1(t-\tau)|,$$

when both f_A and f_B have the same sign, and $= 2|f_2(t) - f_2(t-\tau)|$, when the sign of f_A is different from that of f_B. Therefore, it is seen that the output of the mixer (C) always consists of two different components, $2|f_1(t) - f_1(t-\tau)|$ and $2|f_2(t) - f_2(t-\tau)|$, which are twice the coarse and fine FM, respectively.

If the following relationship can be realized,

(7) $$\frac{K_1 T_1}{2} \geq |f_1(t) - f_1(t-\tau)| > \frac{K_2 T_2}{2} \geq |f_2(t) - f_2(t-\tau)| \quad ,$$

(where $K_1\ T_1/2$ is the maximum frequency shift of the coarse FM and $K_2 T_2/2$ is that of the fine FM) then the separation between coarse and fine FM of the output of mixer (C) can be achieved by using two band-pass filters; one, the fine channel filter, passes the frequency band of zero to $K_2 T_2/2$ (or f_i to $f_i + K_2 T_2/2$ if processed at an intermediate frequency) and the other, the coarse filter, passes the frequency band $K_2 T_2/2$ to $K_1 T_1/2$ (plus f_i if present). If the mixers (A) and (B) differ from the ideal square-law characteristics, and mixer (C) is allowed to produce some unbalanced output, undesired harmonics,

(8) $$\left|(f_1(t) - f_1(t-\tau)) - m(f_2(t) - f_2(t-\tau))\right| ,$$

or

(9) $$\left|n(f_1(t) - f_1(t-\tau)) - (f_2(t) - f_2(t-\tau))\right| ,$$

where m and n are integers larger than 2, may pass through either coarse or fine channels and produce many false outputs. By limiting the maximum frequency shift of the coarse FM as follows:

(10) $$3\,\frac{K_2 T_2}{2} > \frac{K_1 T_1}{2} ,$$

the harmonics given in Eq. (9) can be rejected from the output frequency of mixer (C), although the harmonics given by Eq. (8) will still be present.